Truss us Please

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ABSTRACT

As students taking Statics and Mechanics of Materials I, the professor instructed the class to design a balsa wood bridge capable of carrying a load of 1001 [N] and have a minimum performance index (PI) of 30 [1/$]. The team researched different truss designs including the Warren and the Parker. Different components were inspired by the Warren and Parker truss to make a final design. To calculate the internal reactionary forces acting within each member, the team used the method of joints and utilized symbolics in Matlab to solve the system of equations. To verify the force calculations the method of sections was employed. A number of different iterations of the bridge were tested and the truss design was adjusted in the second iteration through optimizing the angles between members using Matlab. Matlab was used to determine the number of cross beams needed in the third iteration. When testing different iterations of the design, the bridge broke multiple times due to shearing stress on the cross beams and once at a joint. The final bridge held a total load of 1760.16 [N] with a PI of 47.6 [1/$] and cost 16.26 [$].

NOMENCLATURE

A	Cross-sectional area [m^2]
F_i	Internal Reactionary Force of a Member [N] (Sections)
h	Height [mm]
L	Load [N]
PI	Performance Index [1/$]
R_i	Internal Reactionary Force of a Member [N] (Joints)
t	Thickness of Member [m]
Z	Force Per Cross Beam [N]
Special Characters	
$\angle$	Angle [-]
$\overline{\text{A}}$	Line [-]
σ	Normal Stress [MPa]
σ_b	Bearing Stress [MPa]
$\sum$	Summation [-]
τ	Shear Stress [MPA]
τ_{ing}	Shearing Stress [MPa]
θ	Theta [degrees]
$\triangle$	Triangle [-]
A	Cross-sectional area [m^2]
d	diameter of bolt 4.1656 [mm]

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INTRODUCTION

Bridges have been an integral part of civilizations for centuries. Based on an article published by Britannica, the first evidence of bridges being built dates back to 2500 BC (Britannica, 2023). As engineering students, understanding how trusses work is important because of how fundamental these structures are to daily life-especially in a city such as Pittsburgh. Historically the Greeks used trusses in roofing, and construction purposes in the Middle Ages (Britannica, 2023). Today trusses are used extensively in a broad range of buildings and bridges. (Steel Construction). Some of the benefits of trusses include the fact that they can span long distances, are lightweight, have reduced deflection, and can support large loads (Steel Construction). Since trusses distribute forces amongst it members, they can be analyzed and studied to determine an optimal bridge design.

While some of the first bridges included arches and suspension, the team's task was to design a bridge supported by trusses. According to Britannica, trusses are a "Structural member usually fabricated from straight pieces of metal or timber to form a series of triangles lying in a single plane." Similar to these first bridges, this project requires the team to engineer a bridge made of balsa wood.

The team decided on designing a bridge inspired by the Warren and the Parker truss. The Warren truss is known for its geometric properties of repeated equilateral triangles, which influenced the team to incorporate two equilateral triangles on both sides of the truss. In addition, the team used the angled top beams from the Parker in the truss design. The bridge must hold 1001 [N] and have a performance index (PI) of at least 30 [1/$].

METHODOLOGY

To calculate the forces and stresses the bridge experiences, the method of joints and the method of sections were used. The method of joints was used to calculate the internal reactionary forces the members experience within the bridge. The equations derived from the method of joints were solved using Matlab symbolics. The method of joints involves analyzing each joint of the truss and calculating the unknown internal reactionary forces of each member. First a global free body diagram was constructed in order to begin finding the reactionary forces. Then equilibrium equations were created for summation of moments and forces going in the x and y directions. To verify the reactionary forces found from the method of joints, the team used the method of sections. The method of sections involves treating the truss as a rigid body. This method uses the same general free body diagram as the method joints. The general free body diagram was used to write the equilibrium equations in the x and y direction and for the moment. To find the forces with this method, a cut was made through each member. The equilibrium equations from the general free body diagram are then used to calculate each reactionary force. It is important to note there are two types of reactionary forces: tension and compression. These are both present in the method of joints and method of sections. Stress results from the force per area, which were calculated using the reactionary forces found from the method of joints.

Angles

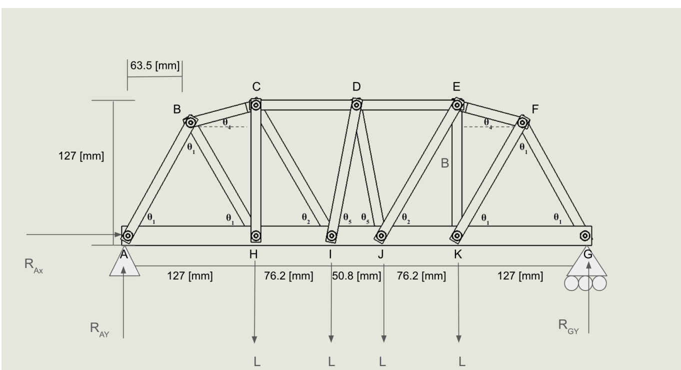

Fig. 1. Global free body diagram of truss design

All of the angles found were in reference to figure 1. The first angle, θ_1, is on the bottom right of $\triangle$ABH. That triangle was made to be equilateral because it was inspired by the Warren truss. θ_1 was found by bisecting $\angle$ABH to turn it into a right triangle. The hypotenuse of the right triangle, $\overline{\mathrm{BH}}$, is equivalent to the height of the bridge as $\triangle$BCH is isosceles. Since the angle value was found using inverse tangent, the length opposite from the angle and the length of the hypotenuse were needed. The length opposite from the angle-the length of the bisector-was found using the Pythagorean theorem, $a^2 + b^2 = c^2$. The adjacent side was then $\frac{1}{2}\overline{\mathrm{AH}}$ or 63.5 [mm]. This means θ_1 was found to be

$$\arctan(\frac{h^2 - 63.5^2}{63.5}) \tag{1}$$

The second angle, θ_2, is the one on the bottom right of $\triangle$CHI. $\overline{\mathrm{CH}}$ is perpendicular to the base of the bridge, therefore $\triangle$CHI is a right triangle. Additionally, $\overline{\mathrm{CH}}$ is the height of the bridge, so the angle of θ_2 is found simply with the equation

$$\arctan(\frac{h}{3}) \tag{2}$$

The third angle was initially defined but was unused, therefore it is not featured in figure 1.

The fourth angle, θ_4, is the angle found in the top left of $\triangle$BCH. θ_4 is not the full angle, $\angle$BCH, but is split by a line that is perpendicular to $\overline{\mathrm{CH}}$. This angle was then found using the bisector made earlier for θ_1 as the height of the new triangle is equal to the height of the bisector subtracted from the total height of the bridge or $h - \sqrt{h^2 - 63.5^2}$. The length of the base of the new triangle is simply $\frac{1}{2}\overline{\mathrm{AH}}$ or 63.5 [mm]. This makes the final calculation of θ_4 to be

$$\arctan(\frac{h - \sqrt{h^2 - 63.5^2}}{63.5}) \tag{3}$$

The fifth angle, θ_5, is the bottom left corner of $\triangle$DIJ. A bisector was made at $\angle$DIJ to find θ_5 since the height of the bisector is the same as the height of the bridge. The base of the new triangle formed then is simply $\frac{1}{2}\overline{\mathrm{IJ}}$ or 12.7 [mm]. This makes the final calculation of θ_5 to be

$$\arctan(\frac{h}{12.7}) \tag{4}$$

Method of Joints

Method of joints is a way to solve for unknown internal reactionary forces acting within the members of a truss. As previously stated, the method of joints requires a set of global equations to be constructed. The global equations solve for the reactionary forces at the pin and roller supports. Pin supports allow a member to rotate but not translate in any direction, while roller supports can rotate and translate along the surface that the roller sits upon (MIT, 1995).

The equations used to solve for the reactionary forces were based upon the assumption that everything is static, therefore the summation of forces in the x axis y axis must equal 0 [N], while the and moments about pin A must equal 0 [Nm]. The positive x axis is towards the right, and the positive y axis is upwards. In figure 1, joint A has a pin support; therefore, joint A has reactionary forces acting in the positive x and positive y direction. Joint G has a roller support; therefore, it has a reactionary force acting in the y direction. The load is assumed to be distributed evenly across joints H, I, J, and K. The team completed all calculations with the total load being 1112 [N], therefore the load at each joint is the total load divided by 2 (number of total trusses) and 4 (number of load carrying joints). The load resulted in 139 [N]. That being said, the global free body diagram equations are as listed below.

$$\sum F_x = R_{Ax} = 0[N] \tag{5}$$

$$\sum F_y = R_{Ay} + R_{Gy} - 4L = 0[N] \tag{6}$$

$$\sum M_A = R_{Gy}(457.2)\,[\text{mm}] - L(127 + 203.2 + 254 + 330.2)\,[\text{mm}] = 0\,[\text{N-m}] \tag{7}$$

R_{Ax}= 0 [N]
R_{Ay}= 278.01 [N]
R_{Gy}= 278.01 [N]

Solving for the unknown reactionary forces in the members was a repetitive process that utilized the equilibrium equations. The equilibrium equations assume the summation of forces in the x axis and y axis equal zero. It is important to understand that when analyzing a joint, the magnitude of the internal reactionary force is assumed to be towards the joint except for the load, and reactionary forces caused by a roller or pin support. Since the truss is symmetric, it can be assumed the reactionary forces will be the same for the corresponding members, therefore only the equilibrium equations at joint A, B, C, D, H and I will be discussed. To solve for the reactionary forces it was important to keep the number of unknowns equal to or less than the number of equilibrium equations.

All the equations will be written in reference to figure 1 to determine which members meet at the joints. A positive reactionary force using the method of joints indicates the member is in compression and a negative reactionary force means the member is in tension.

Joint A has four reactionary forces. Two of those reactionary forces are from the pin support. The other two reactionary forces are due to the two members that connect there. Since the force in member AB is angled, the equilibrium equations must reflect this in the x and y components. The equilibrium equations and reactionary force diagram at joint A are listed below.

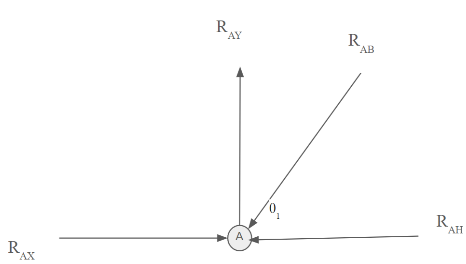

Fig. 2. Free body diagram at joint A.

$$\sum F_x = R_{Ax} - R_{AB}\cos(\theta_1) - R_{AH} = 0[N] \tag{8}$$

$$\sum F_y = R_{Ay} - R_{AB}\sin(\theta_1) = 0[N] \tag{9}$$

R_{AB}= 321.02 [N]
R_{AH}= -160.51 [N]

Moving onto joint B, there are three reactionary forces due to the three members that connect there. Since all of these forces are angled, the equilibrium equations must account for the x and y components of all three members. The equilibrium equations and reactionary force diagram at joint B are listed below.

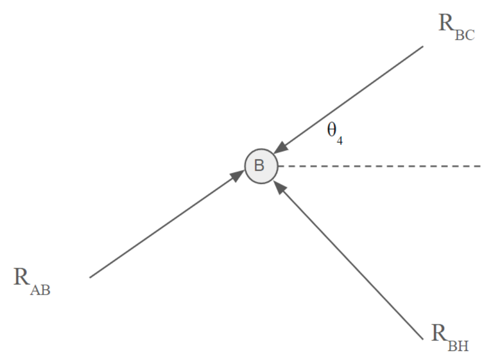

Fig. 3. Free body diagram at joint B.

$$\sum F_x = R_{AB}\cos(\theta_1) - R_{BC}\cos(\theta_4) - R_{BH}\cos(\theta_1) = 0[N] \tag{10}$$

$$\sum F_x = R_{AB}\sin(\theta_1) - R_{BC}\sin(\theta_4) + R_{BH}\sin(\theta_1) = 0[N] \tag{11}$$

R_{BC}= 287.82 [N]
R_{BH}= -235 [N]

Continuing on to joint H, there are four reactionary forces due to the four members that connect there and one load. Forces in members AH, BH, and HI are angled, therefore it is necessary to account for the x and y components when formulating the equilibrium equations. The equilibrium equations and reactionary force diagram at joint H are listed below.

$$\sum F_x = R_{AH} - R_{HI} + R_{BH}\cos(\theta_1) = 0[N] \tag{12}$$

$$\sum F_Y = R_{BH}\sin(\theta_1) + R_{CH} + L = 0[N] \tag{13}$$

R_{CH}= 64.51 [N]
R_{HI}= -278.01 [N]

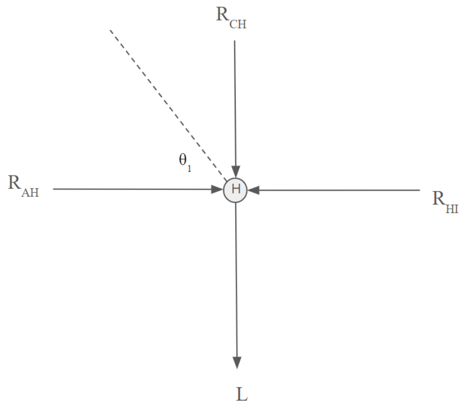

Fig. 4. Free body diagram at joint H.

For joint C, there are four reactionary forces due to the four members that connect there. The equilibrium equations for members BC and CI must account for the x and y force components since they are angled. The equilibrium equations and reactionary force diagram at joint C are listed below.

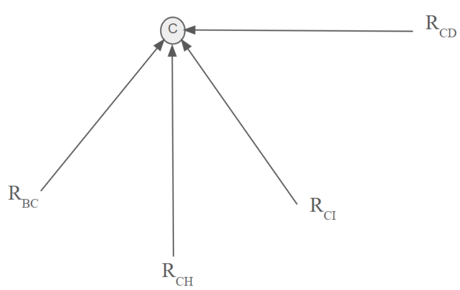

Fig. 5. Free body diagram at joint C.

$$\sum F_x = R_{BC}\cos(\theta_4) - R_{CI}\cos(\theta_2) - R_{CD} = 0[N] \quad (14)$$

$$\sum F_y = R_{BC}\sin(\theta_4) + R_{CI}\sin(\theta_2) + R_{CH} = 0[N] \quad (15)$$

R_{CD}= 361.42 [N]
R_{CI}= -162.11 [N]

Joint I has four reactionary forces due to the four members that connect there and one load. To account for the angled nature of CI and DI, the calculations included the x and y components of these forces. The equilibrium equations and reactionary force diagram at joint I are listed below.

$$\sum F_x = R_{HI} - R_{IJ} + R_{CI}\cos(\theta_2) - R_{DI}\cos(\theta_5) = 0[N] \quad (16)$$

$$\sum F_y = R_{CI}\sin(\theta_2) + R_{DI}\sin(\theta_5) + L = 0[N] \quad (17)$$

R_{DI}= 0 [N]
R_{IJ}= -278.02 [N]

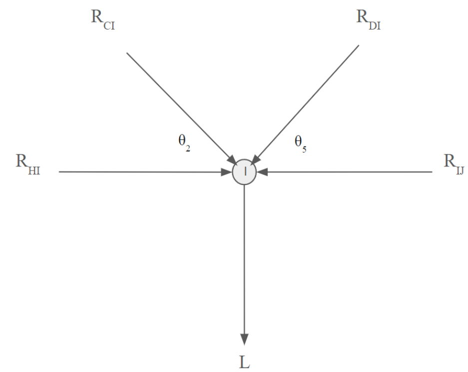

Fig. 6. Free body diagram at joint I.

Lastly, joint D has four reactionary forces due to the four members that connect there. Due to the fact that members DI and DJ are angled, the equilibrium equations must reflect this by accounting for the x and y force components. The equilibrium equations and reactionary force diagram at joint D are listed below.

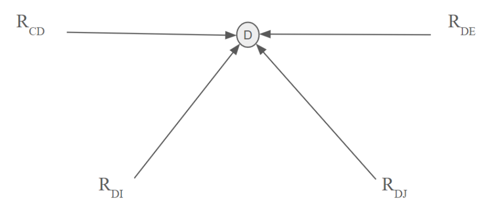

Fig. 7. Free body diagram at joint D.

$$\sum F_x = R_{CD} - R_{DE} + R_{DI}\cos(\theta_5) - R_{DJ}\cos(\theta_5) = 0[N] \quad (18)$$

$$\sum F_y = R_{DI}\sin(\theta_5) + R_{DJ}\sin(\theta_5) = 0[N] \quad (19)$$

R_{DJ}= 0 [N]
R_{DE}= 361.42 [N]

The remaining equations are written in a similar fashion, the equations differ by the joint being analyzed, different angles, different members of the truss, and global forces to use if applicable. In total there were 22 equations (two per joint). Matlab symbolics was utilized to solve the system of equations. Like previously mentioned, the angles were defined in terms of the height; therefore, they were casted into Matlab the same way. The equations were put into Matlab symbolics and an equation solver was used to find the reactionary forces. The results are presented in the table below.

Member	Result [N]	Tension or Compression
AB	321.02	Compression
AH	-160.51	Tension
BC	287.82	Compression
BH	-235.00	Tension
CD	361.42	Compression
CH	64.51	Compression
CI	-162.11	Tension
DE	361.42	Compression
DI	0.00	Compression
DJ	0.00	Compression
EF	287.82	Compression
EJ	-162.11	Tension
EK	64.51	Compression
FG	321.02	Compression
FK	-235.00	Tension
GK	-160.51	Tension
HI	-278.01	Tension
IJ	-361.42	Tension
JK	-278.01	Tension

Method of Sections

The method of sections is another way to solve for unknown reactionary forces acting within members of a truss. To verify the reactionary forces found from the method of joints, the team utilized the method of sections. Both methods use the same free body diagram; therefore, they have the same three global free body diagram equations. Excluding the load and reactionary forces caused by a roller or pin support, the method of sections assumes the magnitude of the reactionary force is pointed away from the cut of the member. When making a section, all forces must be accounted for when formulating the equilibrium equations. When making the cuts, it is important that the cuts are straight lines through members and not joints. The summation of forces in the x and y axis and the moment about pin A must equal zero. This is because the equilibrium equations assume that the body is static. When making a section, all forces must be accounted for when formulating the equilibrium equations. It is important to note that there cannot be more than three unknowns at a time for method of sections; however for the sake of simplicity, the team decided to only have two unknowns. Therefore, the employment of the moment equation per cut will not be necessary.

Solving for the forces within a member using the method of sections is a repetitive process. Since the truss is symmetric, only cuts through the left side of the bridge will be discussed; however, the process was repeated on the right side. A negative reactionary force using the method of joints indicates the member is in compression and a positive reactionary force means the member is in tension.

The first cut was made through members AB and AH. This section of the bridge has reactionary forces at pin A, force AB, and force AH. Due to member AB being angled, the equilibrium equations must account for the x and y force components. The section diagram and equilibrium equation for cut AB and AH are listed below.

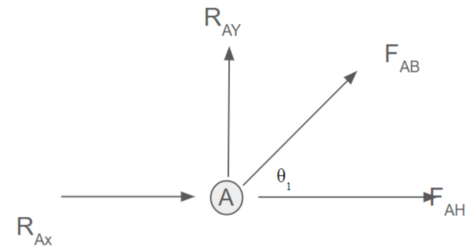

Fig. 8. Free body diagram of cut AB and AH.

$$\sum F_x = R_{Ax} + F_{AH} + F_{AB}\cos(\theta_1) = 0[N] \tag{20}$$

$$\sum F_y = R_{Ay} + F_{AB}\sin(\theta_1) = 0[N] \tag{21}$$

F_{AB}= -321.02 [N]
F_{AH}= 160.51 [N]

For the section created through cutting of members AH, BC, and BH, there are the reactionary forces at pin A, and forces from AH, BC, and BH. Since the forces from members BC and BH are angled, the x and y force components must be included in the equilibrium equations. The section diagram and the equilibrium equations for cut AH, BC, and BH are listed below.

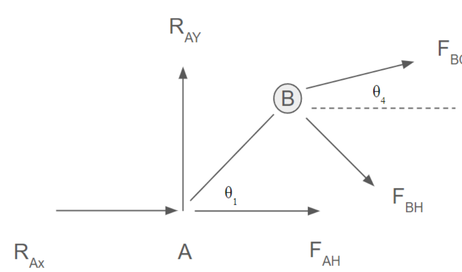

Fig. 9. Free body diagram of cut AH, BC, and BH.

$$\sum F_x = R_{Ax} + F_{AH} + F_{BH}\cos(\theta_1) + F_{BC}\cos(\theta_4) = 0[N] \tag{22}$$

$$\sum F_y = R_{Ay} - F_{BH}\sin(\theta_1) + F_{BC}\sin(\theta_4) = 0[N] \tag{23}$$

F_{BC}= -287.82 [N]
F_{BH}= 235.00 [N]

The section created from the cut through members BC, CH, and HI has a load, reactionary forces from pin A, and forces from BC, CH, and HI. The forces from members BC, and HI are angled therefore the proper x and y components must be accounted for when constructing the equilibrium equations. The section diagram and the equilibrium equations for cut BC, CH, and HI are listed below.

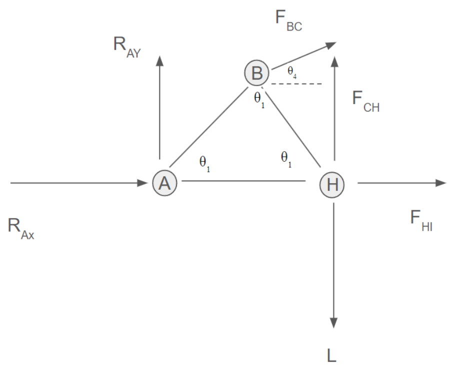

Fig. 10. Free body diagram of cut BC, CH, and HI.

$$\sum F_x = R_{Ax} + F_{HI} + F_{BC}\cos(\theta_4) = 0[N] \quad (24)$$

$$\sum F_y = R_{Ay} + F_{BC}\sin(\theta_4) + F_{CH} - L = 0[N] \quad (25)$$

F_{CH}= -64.51 [N]
F_{HI}= 278.01 [N]

For the section created from the cuts through members CD, CI, and HI, there are the reactionary forces at pin A, a load, and forces from members CD, CI, and HI. The force from member CI is angled therefore the proper x and y components must be accounted for when creating the equilibrium equations. The section diagram and the equilibrium equations for the cut CD, CI, and HI are listed below.

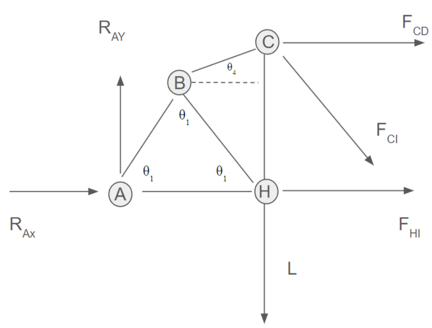

Fig. 11. Free body diagram of cut CD, CI, and HI..

$$\sum F_x = R_{Ax} + F_{HI} + F_{CD} + F_{CI}\cos(\theta_2) = 0[N] \quad (26)$$

$$\sum F_y = R_{Ay} - F_{CI}\sin(\theta_2) - L = 0[N] \quad (27)$$

F_{CD}= -361.42 [N]
F_{CI}= 162.12 [N]

Moving on to the section created by cuts through members CD, DI, and IJ, there are the reactionary forces at pin A, two loads and forces from members CD, DI, and IJ. The force from member DI is angled therefore the proper x and y components must be accounted for when constructing the equilibrium equations. The section diagram and the equilibrium equations for the cut CD, DI, and IJ are listed below.

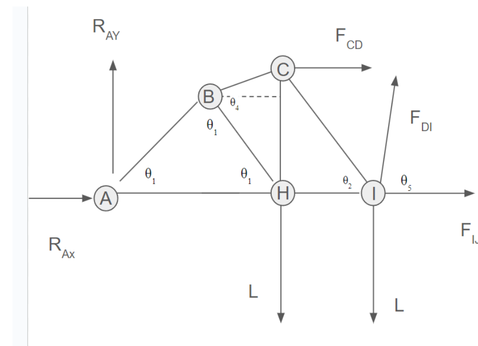

Fig. 12. Free body diagram of cut CD, DI, and IJ..

$$\sum F_x = R_{Ax} + F_{IJ} + F_{CD} + F_{DI}\cos(\theta_5) = 0[N] \quad (28)$$

$$\sum F_y = R_{Ay} + F_{DI}\sin(\theta_5) - 2L = 0[N] \quad (29)$$

F_{DI}= 0.00 [N]
F_{IJ}= 361.42 [N]

Finally, the section created by the cut through members DE, DJ, and IJ, there are the reactionary forces at pin A, two loads, and forces from members DE, DJ, and IJ. The force in member DJ is angled therefore the proper x and y components must be accounted for when constructing the equilibrium equations. The section diagram and the equilibrium equations for the cut DE, DJ, and IJ are listed below.

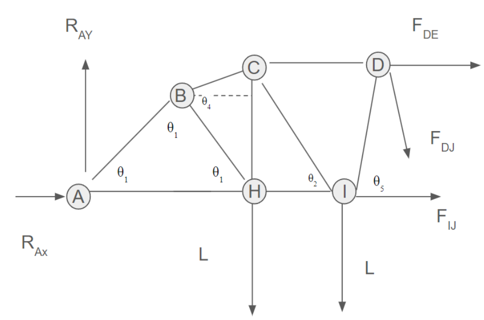

Fig. 13. Free body diagram of cut DE, DJ, and IJ.

$$\sum F_x = R_{Ax} + F_{IJ} + F_{DE} + F_{DJ}\cos(\theta_5) = 0[N] \quad (30)$$

$$\sum F_y = R_{Ay} + F_{DJ}\sin(\theta_5) - 2L = 0[N] \quad (31)$$

F_{DE}= -361.41 [N]
F_{DJ}= 0.00 [N]

The remaining equations were written in a similar manner, the equations differ by the section being analyzed. In total there were 19 equations and the results are presented in the table below.

Member	Result [N]	Tension or Compression
AB	-321.024	Compression
AH	160.510	Tension
BC	-287.822	Compression
BH	235.004	Tension
CD	-361.418	Compression
CH	-64.513	Compression
CI	162.106	Tension
DE	-361.418	Compression
DI	0	Compression
DJ	0	Compression
EF	-287.822	Compression
EJ	162.106	Tension
EK	-64.513	Compression
FG	-321.024	Compression
FK	235.004	Tension
GK	160.510	Tension
HI	278.014	Tension
IJ	361.418	Tension
JK	278.014	Tension

After verifying the internal reactionary forces acting within the members was correct, the team needed to determine which members to rabbet. The team was required to use rabbet joints because of the length of the bolt. The longest bolt length allowed was 38.1 [mm]. The dimensions of the balsa wood was 9.525 [mm] x 9.525 [mm], therefore four joints at a member would exceed the bolt length. Only two out of the four members got rabbeted. To decide which members to rabbet, the team analyzed the internal reactionary forces. If a member was in compression, then the team decided to use rabbet joints. These members were rabbeted because the compressive member pushes towards the bolt. Members BC, CD, DE, and EF got rabbeted together.

Stresses

Before the discussion of stresses begins, it is important to know the three different types of members within the truss. The members in the main beam are AH, HI, IJ, JK, GK. Rabbeted members are BC, CD, DE, and EF. Un-rabbeted members are AB, BH, CH, CI, DI, DJ, EJ, EK, FK, and FG. The type of member impacts the stress calculations because the members have different dimensions.

There are four different types of stress the bridge experiences: normal, shear, shearing, and bearing stress. Normal, shear, and bearing stresses are used for analyzing the stresses within a member while shearing stress is for analyzing the stress within the cross beams. According to an article published by Boston University Mechanical Engineering, normal stress is “When a force acts perpendicular to the surface of an object” (Boston University). The equation is listed below.

$$\sigma = \frac{R}{A} \tag{32}$$

R represents the internal reactionary force of the member being analyzed. A is the cross sectional area of the member being analyzed. Again, there are three different cross sectional areas and these values are presented in the table below.

Main Beam	Rabbeted Member	Not Cut Member
181.45125 [mm^2]	45.36 [mm^2]	90.73 [mm^2]

A design constraint was ensuring the normal stress for members in tension remained under 19.9 [MPa] while members in compression remained under 12.1 [MPa]. Based on the same article published by Boston University, shear stress is “When a force acts parallel to the surface of an object, it exerts a shear stress” (Boston University). The equation is listed below.

$$\tau = \frac{R}{2A} = \frac{\sigma}{2} \tag{33}$$

Since R and A are representative of the same variables as the normal stress, the shear stress is simply half of the normal stress. The shear stress needed to remain under 6.05 [MPa]. Bearing stress is defined as ”The stresses developed when two elastic bodies are forced together” according to an article published by the Engineering Library (Engineering Library). The equation is listed below.

$$\sigma_b = \frac{R}{td} \tag{34}$$

Again, R represents the internal reactionary force of the member being analyzed. d is the diameter of the bolt and t is the thickness of the member. The diameter of the bolt is constant, at 4.1656 [mm]. The bearing stress should remain under 12.1 [MPa]. It is important to note, there are two different thicknesses, these values are presented in the table below.

Main Beam	Rabbeted Member	Not Cut Member
9.53 [mm]	4.76 [mm]	9.53 [mm]

With that being said, the results for normal, shear, and bearing stress are presented below in the table.

Member	Normal [MPa]	Shear [MPa]	Bearing [MPa]
AB	3.53	1.765	8.09
AH	0.884	0.442	4.045
BC	6.345	3.1725	14.508
BH	2.59	1.295	5.92
CD	7.967	3.9835	18.217
CH	0.71	0.355	1.626
CI	1.787	0.8935	4.086
DE	7.967	3.9835	18.217
DI	0	0	0
DJ	0	0	0
EF	6.345	3.1725	14.508
EJ	1.787	0.8935	4.086
EK	0.71	0.355	1.626
FG	3.53	1.765	8.09
FK	2.59	1.295	5.92
HI	1.532	0.766	7.01
IJ	1.992	0.996	9.109
JK	1.532	0.766	7.01
KG	0.884	0.442	4.045

Although the bearing stress for members BC, CD, DE, and EF exceeded the constraint, the team proceeded with the design out of curiosity if the math would perfectly translate to the real world (It did not). Lastly, shearing stress acts co-planar to the cross section of the material (Xometry). In this bridge this stress acts in the cross beams. The equation is listed below.

$$\tau_{ing} = \frac{Z}{A} \tag{35}$$

Z is representative of the total load divided by 2 (number of trusses) and 6 (number of crossbeams). Therefore Z is 92.67 [N], again this number was found with the assumption that the total load is 1112 [N]. A is representative of the cross sectional area of the cross beams, which is conveniently the same as the cross sectional area of a not cut member. According to an article published by the Massachusetts Institute of Technology, the maximum shearing stress for balsa wood is 5 [MPa] (MIT Libraries, 2015). The shearing stress was found to be 1.02 [MPa].

Optimization

After testing the first bridge design, which failed due to shearing, the team decided to calculate the exact number of cross members needed to carry the expected load of the bridge. The Matlab code that was created to calculate the internal reactionary forces for the method of joints was used as the base of the program to calculate the maximum weight the bridge could hold. The stresses of each member were then calculated at the bottom of this code and the code was run in a loop, increasing the weight by approximately 22 [N] with each pass through. Additionally, with each run, the stresses were compared to the maximum stress the balsa wood could withstand before breaking. The loop would then break when the code found that one of the stresses in the bridge was equal to or greater than the maximum stress. It's important to note that bearing stress already started above the maximum allowable stress; therefore, it was not included in the loop. This decision was ultimately made given off the previous test run in which, with the weight held, the bearing stress should have doubled that of which was allowable and yet the pieces still did not fracture. Through this method, it was found the initial design would be expected to hold approximately 1668 [N]. This weight was then plugged back into the shearing stress calculations and it was found that the number of cross members would need to be at least eight.

Additionally, in an attempt to evenly distribute the load across members AB, BC, and CD, a Matlab script was made to optimize the angles based on the height of the bridge. Due to the symmetry of the truss, it is expected that members DE, EF, and FG would react the same as their corresponding members. This was done by maintaining the same distance between the attachments on the base of the truss and making the angles variable based on the height of the bridge.

After examining the geometry of the bridge, it was decided that the code should start with a height of 65.0875 [mm] in order to avoid having the code attempt to run geometries that can't exist in real life. Specifically, in reference to equation one, if a height of 63.5 [mm] were to be run, θ_1 would be 0. Therefore, implying the members would have to be parallel to the base and would no longer form a triangle, but a line. The code increased the height with every pass by 1.5875 [mm] up to 254 [mm]. The graph of the force on the members vs the height is shown in the figure below. This graph was then

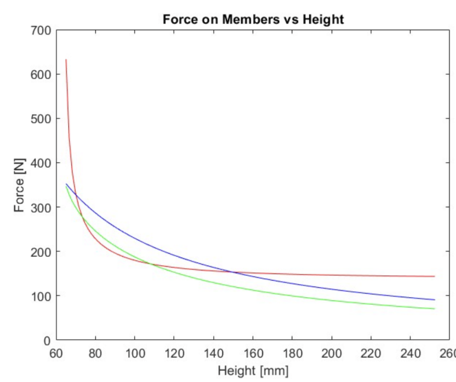

Fig. 14. Forces on 3 main members, AB - red, BC - green, CD - blue .

analyzed to see if there was ever a point in which all the forces were exactly the same. Since there was no such point, the next best thing was to find the point in which the forces were the closest together. It was then determined that the height where the forces were the closest was 73.025 [mm]. Once the height and angles were determined, they were used to calculate the distance of the holes. These distances were then used to create the second bridge.

Performance Index (PI)

To calculate the PI, the load is divided by the weight of the bridge plus the cost. The cost is composed of the cost of materials in addition to the environmental cost. The total environmental cost was \$0.7014 while the cost of the materials was \$15.5586 total. One 9.525 [mm] x 19.05 [mm] x 1219.2 mm beam was used, which had a material cost of \$2.40 and an environmental cost of \$0.0672 for a total cost of \$2.4672. Approximately, four 9.525 [mm] x 9.525 [mm] x 1219.2 [mm] beams were used, which had a material cost of \$7.6228 and an environmental cost of \$0.1334, for a total cost of \$7.7562. The bridge used four 25.4 [mm] bolts which had a material cost of \$0.36 and an environmental cost of \$0.0476, for a total cost of \$0.4076. In addition, eighteen 38.1 [mm] bolts were used which had a material cost of \$2.52 and an environmental cost of \$0.2952, for a total cost of \$2.8152. Forty-four total washers were used, which had a material cost of \$1.76 and an environmental cost of \$0.0484, for a total cost of \$1.8084. Finally, there were twenty-two nuts that had a material cost of \$0.88 and an environmental cost of \$0.1254, for a total cost of \$1.0054. Together the costs add up to a total of approximately \$16.26. Referenced below is the total cost breakdown for each millimeter of material provided by the project description.

Item	Item Cost [\$/Item]	CO_2 Cost [\$/Item]
9.525 x 19.05 [mm^2]	0.001969	0.000055
9.525 x 9.525 [mm^2]	0.00157	0.000028
No 8 Washer	0.04	0.0011
8/32 nut	0.04	0.0057
25.4 [mm] 8/32 bolt	0.09	0.0119
38.1 [mm] 8/32 blot	0.14	0.0164

The bridge ultimately weighed 2.2736 [N] and held a total load of 1760.16 [N]. With this in mind, the final PI calculation ended up being $\frac{1760.16}{2.2736x16.26}$ or approximately 47.6.

RESULTS AND DISCUSSION

The first iteration of the bridge had 6 cross members and held 1272 [N] with a PI of 35.9 [1/\$]. The initial hypothesis was that the rabbet joints would be the first to break due to bearing stress. This hypothesis was drawn from comparing the stress calculations to the maximum allowable stress. However, after testing it was found that the cross members of the bridge broke first. This implied the bridge broke due to shearing stress rather than bearing stress. To improve upon the initial design, two different plans were devised.

The first plan was to evenly distribute the force on the bridge as much as possible by optimizing the angles of the bridge based on the height of the truss. However, by distributing the load evenly across all members, the internal reactionary forces were increased on the rabbet joints. This resulted in an increase of stress on the rabbet joints. This resulted in the failure of the bridge in member AB at a much lower force than the initial bridge design. This second bridge design decreased the height of the bridge by 50 [mm], had 8 cross members, held 787 [N], and had a PI of 34.2 [1/\$].

The other plan was to calculate the maximum weight the bridge could hold based on the stresses. The maximum weight was then used to calculate the necessary number of cross members for the first truss design. The theoretical maximum weight was calculated to be 1668 [N] and the minimum number of cross members needed was calculated to be eight. This theory was tested by re-using the truss from the first iteration and gluing on new cross members. With eight cross members the bridge was able to withstand 1414.5 [N] and have a PI of 37.5 [1/\$]. Once again, the main point of failure of the bridge was the shearing stress. Therefore, ten cross members were used in the final design.

In the final test, the main beam split at joint K. Additionally, member FK broke due to bearing at joint K. All the cross members stayed intact for the final test. The final bridge held a force of 1760.16 [N] with a PI of 47.6 [1/\$] and cost 16.26 [\$].

CONCLUSION

The Professor assigned his Statics and Mechanics of Materials I class with the challenge of designing a balsa wood bridge capable of supporting a load of 1001 [N] and a minimum PI of 30. After researching different truss designs, the team decided to draw inspiration from the Warren and Parker truss for the final design. To determine the internal reactionary forces acting within the members of the truss, the team leveraged symbolics in Matlab to solve the system of equations created from the method of joints. The method of sections was used to validate the forces found from the method of joints. While iterating the truss design, Matlab was utilized to optimize the number of cross beams needed and the angles used within the truss. After numerous tests, the final bridge design held a load of 1760.16 [N] had a PI of 47.6 [1/\$] and cost 16.26 [\$].

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