Designing of a Bi-Metallic Strip for a Thermal Switch

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OBJECTIVE

The objective of the project is to design a bi-metallic strip that is 3 [in] in length with a fixed end. The bimetallic strip designed is constrained such that the deflection is less than 0.05 [in] when there is a change of temperature of 50°F.

NOMENCLATURE

Symbols	Definition	Units
α	Coefficient of Thermal Expansion	[-]
A	Cross Sectional Area	$[in^2]$
h	Height	[in]
E	Elastic Modulus	[Pa]
L	Length	[in]
T	Temperature	[°C]
y	Allowable Deflection	[in]
Δx	Change in length	[in]
F	Force	[N]
n	Ratio of Elastic Moduli	[-]
I	Second Moment of Area	$[in^4]$

TABLE I
SYMBOLIC DEFINITIONS

BACKGROUND

Bi-metallic strips are devices that deflect due to changes in temperature. Typically used in thermometers and other temperature sensitive measurement devices, bi-metallic strips are made up of 2 different metals attached to each other lengthwise. When the strip experiences a change in temperature, each individual strip will expand accordingly to the coefficient of thermal expansion. Due to the adhesive firmly attaching the two strips together, the strips must remain the same length. Thus, one strip will be in tension and the other in compression causing a couple to form at the end of bi-metallic strip resulting in an observed deflection. As the temperature change increases, the thermal deformation, and the couple will increase resulting in a more noticeable deflection. This process can be seen below in figure 1.

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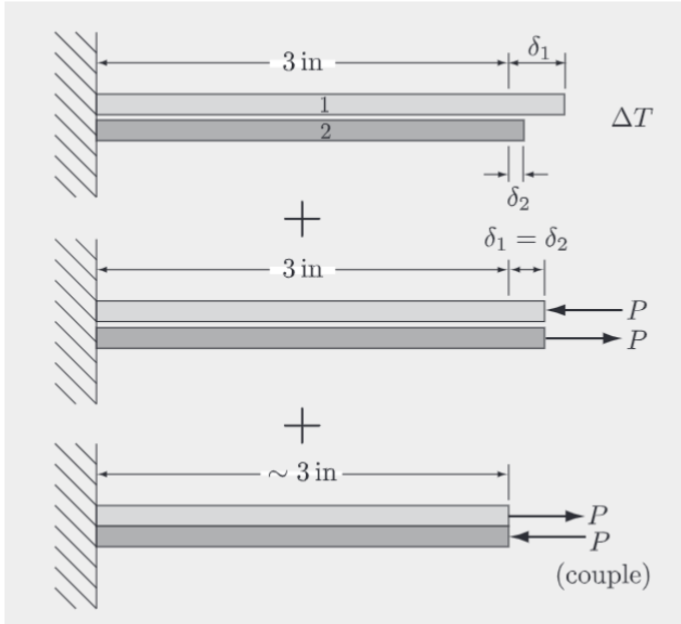

Fig. 1. Mechanics of a Bi-Metallic Strip

METHODOLOGY

In this project, the heights of each strip, the base, and the material are all variable. In order to complete the objective, these parameters must be carefully selected such that the bi-metallic strip deflects less than 0.05 [in] due to the 50°F temperature change. This was accomplished by deriving an equation that related all of these parameters to the beam's deflection.

To properly analyze the deflection, the bi-metallic strip was represented as a cantilevered beam with a moment at the end. This is referenced in Case 4 which states that a cantilevered beam with a moment located on the opposing end of the support can be represented by:

$$y = \frac{ML^2}{2EI} \tag{1}$$

The steps and derivation for each component in this equation are outlined below.

Assumptions

Before any analysis was completed, a list of assumptions was made to make the analysis simpler. They are listed below:

1) The Coefficient of Thermal Expansion for material 1 is greater than that of material 2. ($\alpha_1 > \alpha_2$)
2) The heights of both materials are the same. ($h_1 = h_2 = h$)

Moment

The first step in this analysis took into account the different lengths due to thermal expansion. The general thermal expansion equation is modeled below as:

$$\Delta x = \alpha L \Delta T \tag{2}$$

Using this equation, the change in lengths for each metal is modeled such that:

$$\Delta x_1 = \alpha_1 L \Delta T \tag{3}$$

$$\Delta x_2 = \alpha_2 L \Delta T \tag{4}$$

For this analysis, the difference between the thermal expansion of the metals is more important than the magnitude of the thermal expansion. Therefore, the difference between the thermal expansions is defined as Δx such that:

$$\Delta x = \Delta x_1 - \Delta x_2 = L \Delta T(\alpha_1 - \alpha_2) \tag{5}$$

Deflection Equations

In a bi-metallic strip the two strips are the same length despite having two different metals with two different rates of expansion. This is only possible if there is an equal and opposite force acting on each strip such that one strip is compressed and the other strip under tension. This can be modeled by the general deflection equation which is shown below:

$$\delta = \frac{FL}{EA} \tag{6}$$

F is the reaction force. L is the length of the strip. E is the elastic modulus and A is the cross-sectional area.

1) strip 1: As stated in the assumptions, $\alpha_1 > \alpha_2$, therefore, strip 1 will expand more than strip 2. Thus, strip 1 will need to be compressed or have a negative deflection. From inspection:

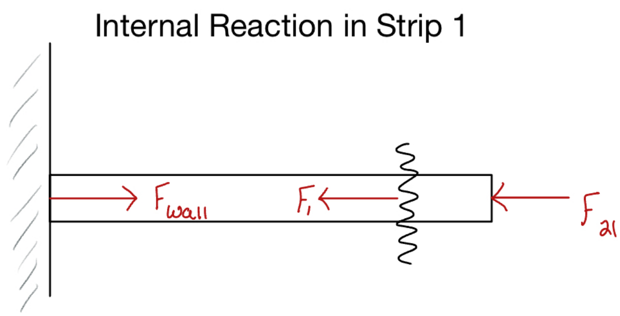

Fig. 2. Internal Reaction Force of Strip 1

$$\Sigma F_x = F_w - F_1 = 0; F_1 = F_w \tag{7}$$

2) Strip 2: As stated in the assumptions, $\alpha_1 > \alpha_2$, therefore, strip 2 will expand less than strip 1. Thus, strip 2 will be in tension. From inspection:

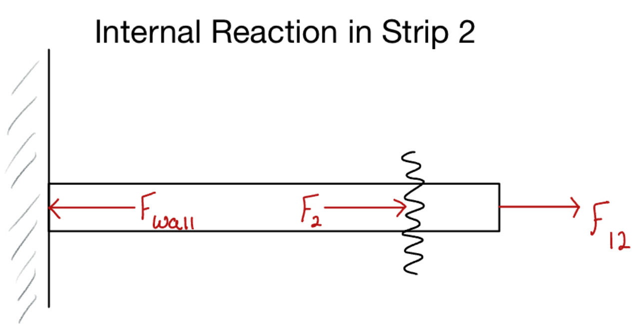

Fig. 3. Internal Reaction Force of Strip 2

$$\Sigma F_x = -F_w + F_2 = 0; F_2 = F_w \tag{8}$$

Combining these together, it can be said that

$$F_w = F_1 = F_2 = F \tag{9}$$

As stated earlier, the two strips are the same length. Thus, the total expansion due to thermal expansion and axial loading must be the same for both strip 1 and strip 2. This can be represented below as:

$$\Delta x_1 - \delta_1 = \Delta x_2 + \delta_2 \tag{10}$$

Which can then be further simplified into:

$$\Delta x = \frac{FL}{E_2 A_2} + \frac{FL}{E_1 A_1} \tag{11}$$

And solving for force will yield:

$$F = \frac{bh\Delta T(\alpha_1 - \alpha_2)(E_2 + E_1)}{E_1 E_2} \tag{12}$$

Moment and Couple

The force calculated above generates a couple at the end of strip as there are two opposite acting forces separated by a distance. Given the assumption that the two strips are the same height, h, then the distance between the two forces acting on the centroid of the strips can be calculated as:

$$x = y_1 - y_2 = \frac{3h}{2} - \frac{h}{2} = h \tag{13}$$

The couple can then be calculated as:

$$M = Fx = \frac{bh\Delta T(\alpha_1 - \alpha_2)(E_2 + E_1)}{E_1 E_2}(h) = \frac{bh^2\Delta T(\alpha_1 - \alpha_2)(E_2 + E_1)}{E_1 E_2} \tag{14}$$

At this point in the analysis, the strip can be modeled as the diagram shown below:

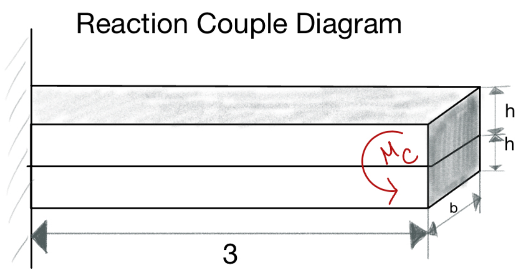

Fig. 4. Couple

Transformed Section and Finding The Second Moment of Area

In order to derive the second moment of area, the bi-metallic strip must be analyzed as if it were a single material. In this analysis, the strip was converted into material 2 as seen in the diagram below:

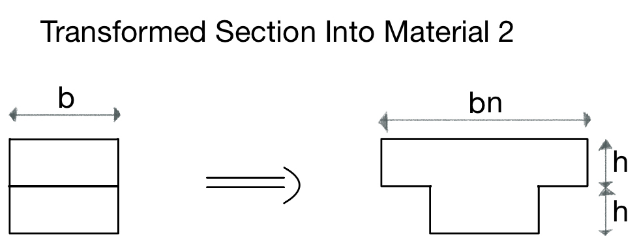

Fig. 5. Transformed Section

In the transformed section, y_c must be found, which includes y_{c1} and y_{c2} being the centroid locations of each material, respectively. In addition, A_{c1} and A_{c2} are the areas of each material. It is important to note that n = $\frac{E_1}{E_2}$.

$$y_{c1} = \frac{3}{2}h; A_{c1} = bnh \tag{15}$$

$$y_{c2} = \frac{1}{2}h; A_{c2} = bh \tag{16}$$

Now that all of the necessary components for y_c are solved, the general equation can be found below:

$$y_c = \frac{y_{c1}A_{c1} + y_{c2}A_{c2}}{A_{c1} + A_{c2}} = \frac{h(3n+1)}{2(1+n)} \tag{17}$$

Since y_c is derived, the equations for the second moment of areas must be solved. Assuming that section 1 is the section that is being transformed, the equations are modeled below:

$$I_1 = \frac{1}{12}(bn)h^3 + (y_{c1} - y_c)^2 A_{c1} = \frac{h^3 bn}{12} + \frac{h^3 bn}{(1+n)^2} \tag{18}$$

$$I_2 = \frac{1}{12}bh^3 + (y_c - y_{c2})^2 A_{c2} = \frac{h^3 b}{12} + \frac{h^3 bn^2}{(1+n)^2} \tag{19}$$

After these unknowns are solved for, the total Second Moment of Area is:

$$I_{tot} = I_1 + I_2 = (bh^3)\frac{n^2 + 14n + 1}{12(1+n)} \tag{20}$$

Combining Equations

Using the derived moment and second moment of area equations, the final allowable deflection equation is solved for:

$$y = \frac{ML^2}{2EI} = \frac{6nL^2 \Delta T(\alpha_1 - \alpha_2)}{h(n^2 + 14n + 1)} \tag{21}$$

Matlab Code

Following the final derivation of the maximum deflection equation, a Matlab code is utilized to test a combination of different metals. The list of metals to analyze was reduced to brass, copper and steel due to their accessibility. To begin, a list of constants (i.e. elastic moduli, and coefficient of thermal expansion) was constructed in reference to articles published by Ezlok, AmesWeb, and the Engineering Tool Box. Those values are listed in the table below.

Material	Elastic Modulus $*10^6$ [psi]	Coefficient of Thermal Expansion $*10^{-6}$[1/F]
Brass	14.1	10.00
Copper	16.7	8.89
Steel	28.00	9.61

There was a possibility of 6 different configurations due to the three different materials. The first combination of materials to be tested is brass and copper. When using the aforementioned method of analysis in which a section is transformed, either material can be transformed, which results in the two sub-cases of: brass transformed to an entirely copper section, and copper being transformed to an entirely brass section. Using assumption 1 helps eliminate a sub-case. It is important to note the same methodology was applied to the remaining 2 main cases (i.e. brass and steel, and copper and steel).

With that being said, equation 21 can be used to solve for the maximum deflection. Within the Matlab script, the height, h, is defined as an array ranging from 0 to 1 [in] being incremented by 0.01 [in]. Three separate plots were made to represent the viable cases. These plots can be seen in the Results and Discussion section. The plots show the Difference in height vs. the Deflection.

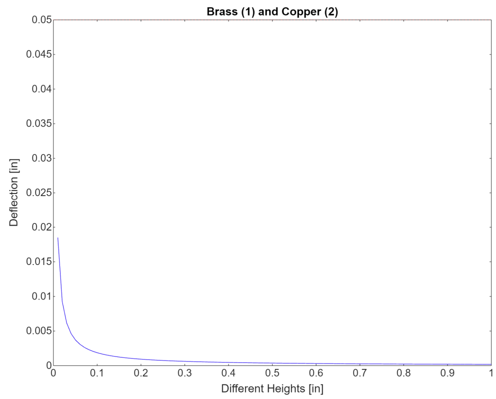

Fig. 6. Different Heights vs. Deflection for Brass and Copper

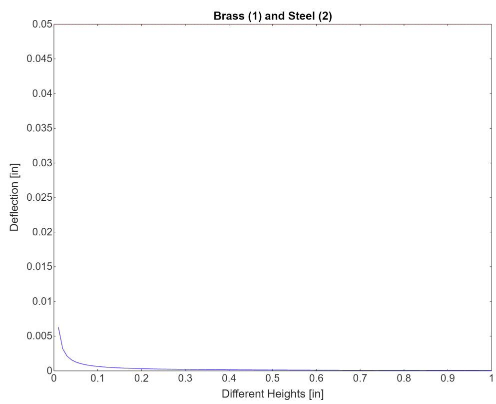

Fig. 7. Different Heights vs. Deflection for Brass and Steel

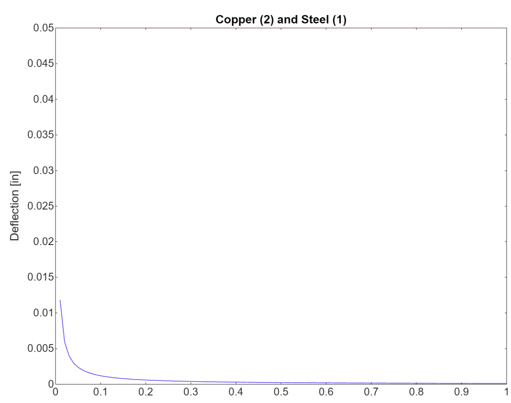

Fig. 8. Different Heights vs. Deflection for Copper and Steel

FINAL RESULTS

As shown in the graphs above, the three cases have similar outcomes when discussing the amount of deflection. As the height of each strip increases, the deflection in each bimetallic strip decreases exponentially. On the contrary, this means as the height of each strip decreases, the deflection will increase exponentially.

While a 2 [in] thick bimetallic strip would theoretically give the best outcome, as it provides the least deflection, this is highly unlikely and unusable in the real world. Due to this fact, the final choice of metals will be based on any thickness less than 0.5 [in]. Narrowing further, a common thickness for each strip is 0.02 [in].

As the height of the strip approaches zero, it is apparent from the graphs that the bimatallic strip made of Brass and Steel deflects the least. Using the previously determined common thickness, 0.02[in], the brass and steel will both be 0.02 [in] thick with a maximum deflection of 0.003 [in].

Although in the calculations, the width of the base did not matter, for practicality reasons, the width will be 0.5 [in].

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