Preventing Our Boiler from Doing Work

Kalinda Wagner*
Mechanical Engineering Student
kaw345@pitt.edu

Nada Elgendy
Mechanical Engineering Student
nee16@pitt.edu

Abboud Tamer
Mechanical Engineering Student
ajt150@pitt.edu

ABSTRACT

As students taking Intro to Thermodynamics, the Professor instructed the class to maximize the thermal efficiency of a traditional steam driven Rankine Cycle. To optimize the system, the pumps and turbines were initially assumed to be isentropic. After defining some state properties, a Matlab script and XSteam were used to find the remaining properties. The 1^{st} and 2^{nd} law of thermodynamics were utilized to analyze the system. The equations got embedded into the script and were used to iterate through different possibilities of the cycle. The efficiency of the pumps and turbines were used to find the real efficiency (i.e. initial assumption regarding pumps and turbines is discarded). The iteration in which the system was optimized had a real efficiency of 27.20 % when P_2 = 4.11 [bar] , P_3 = 3.30 [bar] , and P_5 = 0.88 [bar].

NOMENCLATURE

g	Gravity 9.81 [$\frac{m}{s^2}$]
h	Specific Enthalpy [$\frac{kJ}{Kg}$]
P	Pressure [bar]
q	Specific Heat[$\frac{kJ}{Kg}$]
s	Specific Entropy [$\frac{kJ}{Kg-K}$]
T	Temperature [°C]
u	Energy of the System $\frac{kJ}{kg}$
V	Velocity [$\frac{m}{s}$]
w	Specific Work [$\frac{kJ}{Kg}$]
x	Quality [-]
y	Mass Fraction
z	Height [m]
Special Characters	
η	Efficiency [-]
σ	Entropy Generated [$\frac{kJ}{Kg-K}$]
$\sum$	Summation
Acronyms / Subscripts	
B	Boiler
C.F.H.	Closed Feedwater Heater
C.o.E	Conservation of Energy
E.R.B.	Entropy Rate Balance
H.P.	High Pressure
HEX	Heat Exchanger
L.P.	Low Pressure
O.F.H.	Open Feedwater Heater

*All correspondence regarding the paper should be addressed to Kalinda Wagner

INTRODUCTION

Thermal efficiency is important to designing power plants. It is used to ensure that the system designed can produce the most energy possible with minimal input while staying within the confines of reality. In a publication by Boston University, it states the thermal efficiency of a thermal power plant went from 4% to 439% efficient. This is due to a reduction of heat loss in the boiler, turbine, and engine of power plants. The increase in efficiency has a large effect on lowering cost of electricity and minimizing the use of natural resources [1]. Hence why it is vital to understand and optimize thermal efficiency.

One part of understanding thermal efficiency is understanding the Carnot Cycle. Referencing a Youtube video published by the channel Matt Barry, "The Carnot Cycle is the most efficient heat engine operating between any two temperature reservoirs" [2]. A derivation of the Carnot Cycle, is the Rankine Cycle. An article issued by the University of Calgary states, the Rankine Cycle converts heat into mechanical energy. This is achieved through heating water to produce steam and using the steam to operate turbines which generate work. It is often used in nuclear reactors [3]. Since the Carnot Cycle is not attainable, the Rankine Cycle is a more realistic way of achieving as close to Carnot as possible. Therefore the thermal efficiency of the Rankine Cycle can not exceed that of the Carnot Cycle.

METHODOLOGY

In order to optimize the Rankine cycle, the state properties were identified, initially assuming the pumps and turbines are isentropic. A Matlab script and XSteam were utilized to find values for the remaining state properties. Conservation of energy (C.o.E) and entropy rate balance (E.R.B.) equations were formulated for each component, which were coded into the Matlab script. Lastly, the initial assumption about the pumps and turbines is discarded to account for irreversibilities which increase entropy and enthalpy in any process.

Defining State Properties

The proposed cycle is shown below alongside a list of limits and requirements that were given and used to determine state properties. Each state must have two independent properties defined in order to derive the remaining three. It is important to note that water is the working fluid.

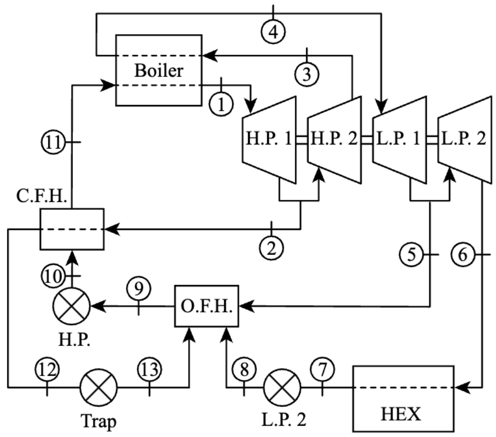

Fig. 1. Combined Cycle Schematic

1) Maximum high-pressure(H.P.) turbine inlet conditions: P = 8.0 [MPa], T = 480°C;
2) Minimum turbine quality at outlet is 90 %;
3) Reheat temperature may not exceed 440°C;
4) Substances entering pumps must be saturated liquid or a compressed / subcooled liquid;
5) The isentropic efficiency of the turbines is 85%;
6) The isentropic efficiency of the pumps is 90%;
7) Heat transfer in all heat exchangers / condensers requires a minimum of 15°C temperature difference between working fluids;
8) The ambient temperature that the condenser is exposed to is 25°C;

The first constraint provides the state properties at state 1. The pressure is 8.0 [MPa], or 80.0 [bar], and a temperature of 480°C. The second constraint provided the minimum quality for states 2, 3, 5, and 6. The quality of any turbine outlet must be greater than 90% or $x \geq 0.9$ [-]. The third constraint limits the reheat temperature coming out of the boiler to be a maximum of 440°C. This was used as the temperature at state 4 for the purposes of maximizing efficiency of the system. The fourth constraint defines the quality at states 7 and 9 to be x = 0.0 [-]. Constraint five and six will be used as boundary conditions and discussed later on. The seventh constraint specifies the minimum difference in temperature between interacting fluids. This is because there has to be a temperature difference in order for the initial substance to be cooled. This implies the temperature at state 11 has to be 15°C less than the temperature at state 2 ($T_{11} = T_2$ - 15 °C). In addition, the boundary temperature for the boiler is found to be 495°C, 15°C greater than the maximum heat output. Constraint eight, states the ambient temperature is 25°C, this gives the temperature at state 7 and the temperature boundary for the heat exchanger (HEX) as 40°C, 15°C greater than ambient, using constraints seven and eight together. Like previously mentioned, the pumps and turbines are initially assumed to be isentropic processes. The trap is isenthalpic, therefore the inlet enthalpy is equivalent to the outlet enthalpy (i.e. $h_{12} = h_{13}$). Additionally, it is recognized that the closed-feedwater heater (C.F.H), open-feedwater heater (O.F.H), HEX and boiler are isobaric processes. Therefore the inlet pressure is equivalent to outlet pressure. This can be visualized in the figure below.

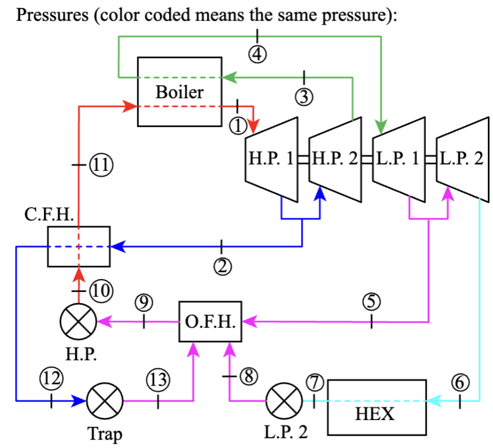

Fig. 2. Pressure Schematic

When analyzing the mass flow rates, if there is a junction in the path (i.e. multiple outlets from one system component) the mass flow will split. This causes a mass fraction. An example of this can be seen at each turbine. Additionally, when there are multiple inlets going into one component of the system, the mass flows are added together. An example of this can be seen at the O.F.H. All of this is represented in the diagram below.

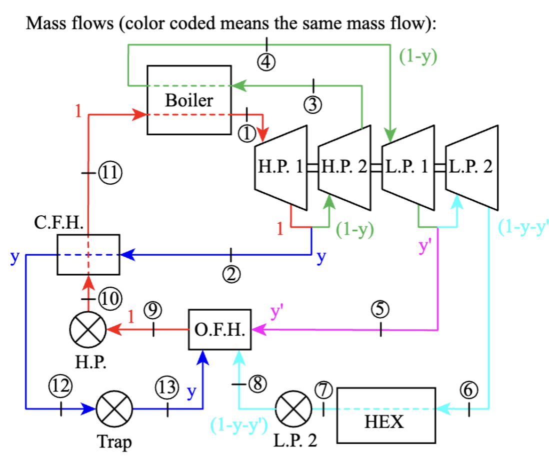

Fig. 3. Mass Flow Schematic

Applying the 1^{st} Law; Conservation of Energy

The base equation for the C.o.E. is shown below

$$u = q - w + \sum_i (h_i + \frac{(V_i)^2}{2} + g * z_i) - \sum_j (h_j + \frac{(V_j)^2}{2} + g * z_j) \quad (1)$$

The 1^{st} law equation states, energy of the system is equivalent to the specific heat subtracted from specific work plus the specific energy exiting subtracted from specific energy entering the system. Note the energy entering or exiting is comprised of three components, specific internal energy, specific kinetic energy and specific potential energy, respectively.

To reduce equation 1, note that kinetic energy and potential energy do not contribute to the energy of the system. Since the overall system is steady state, the energy of the system is 0 $[\frac{kJ}{kg}]$. Again, all turbines and pumps are assumed to be isentropic, therefore the specific heat is equivalent to zero. In addition, the specific heat at the C.F.H., O.F.H. and the trap are assumed to be 0 $[\frac{kJ}{kg}]$ because they are adiabatic. For the isobaric components (i.e. Boiler, C.F.H., HEX, and O.F.H.), the specific work is assumed to be 0 $[\frac{kJ}{kg}]$ as well as the trap. The reduced C.o.E. equations for each component are listed below. Notice the equation at the O.F.H. and the C.F.H. provide definitions for the mass flow rates.

H.P. turbine 1:

$$w_{HP-t1} = h_1 - h_2 \tag{2}$$

H.P. turbine 2:

$$w_{HP-t2} = (1-y)*(h_2 - h_3) \tag{3}$$

Boiler:

$$q_B = (h_1 - h_{11}) - (1-y)*(h_3 - h_4) \tag{4}$$

L.P. turbine 1:

$$w_{LP-t1} = (1-y)*(h_4 - h_5) \tag{5}$$

L.P. turbine 2:

$$w_{LP-t2} = (1-y-y^{'})*(h_5 - h_6) \tag{6}$$

HEX:

$$q_{HEX} = (1-y-y^{'})*(h_7 - h_6) \tag{7}$$

L.P. pump:

$$w_{LP-p} = (1-y-y^{'})*(h_7 - h_8) \tag{8}$$

O.F.H.:

$$y^{'} = \frac{y*(h_3 - h_8) + (h_8 - h_9)}{h_8 - h_5} \tag{9}$$

H.P. pump:

$$w_{HP-p} = h_9 - h_{10} \tag{10}$$

C.F.H.:

$$y = \frac{h_{11} - h_{10}}{h_2 - h_{12}} \tag{11}$$

Trap:

$$h_{12} = h_{13} \tag{12}$$

Applying the 2^{nd} Law; Entropy Rate Balance

The base equation for E.R.B. is shown below

$$\Delta s = \sum \frac{q}{T_b} + \sigma + \sum_i s_i - \sum_j s_j \tag{13}$$

The 2^{nd} law equation states. the change of specific entropy is equivalent to the summation of specific heat divided by the boundary temperature added to the specific entropy generated and the specific entropy entering minus the specific entropy exiting.

To reduce equation 13, note the pumps and turbines are still assumed to be isentropic, therefore the specific heat and specific entropy generated are equivalent to zero. In addition, the specific heat at the C.F.H., O.F.H. and the trap are assumed to be 0 $[\frac{kJ}{kg}]$ because they are adiabatic. Intuitively, the system is optimized when the change in specific entropy is equivalent to 0 $[\frac{kJ}{kg}]$. The reduced E.R.B. equations for each component are listed below. A majority of the equations serve as a mathematical proof for the state properties, since they were initially defined by intuition. For the remaining equations, it is important to note, the specific entropy generated can not be less than zero.

H.P. turbine 1:

$$s_1 = s_2 \tag{14}$$

H.P. turbine 2:

$$s_2 = s_3 \tag{15}$$

Boiler:

$$\sigma_B = \frac{-q_B}{T_B} + (s_1 - s_{11}) + (1-y)*(s_4 - s_3) \tag{16}$$

L.P. turbine 1:

$$s_4 = s_5 \tag{17}$$

L.P. turbine 2:

$$s_5 = s_6 \tag{18}$$

HEX:

$$\sigma_{HEX} = \frac{-q_{HEX}}{T_{HEX}} + (1-y-y^{'})*(s_7 - s_6) \tag{19}$$

L.P. pump:

$$s_7 = s_8 \tag{20}$$

O.F.H.:

$$\sigma_{OFH} = s_9 - y*s_{13} - s_8*(1-y-y^{'}) - y^{'}*s_5 \tag{21}$$

H.P. pump:

$$s_9 = s_{10} \tag{22}$$

C.F.H.:

$$\sigma_{CFH} = \frac{s_{11} - s_{10}}{y*(s_2 - s_{12})} \tag{23}$$

Trap:

$$\sigma_{Trap} = y*(s_{13} - s_{12}) \tag{24}$$

Efficiency

The equation for the overall efficiency of the Rankine cycle is shown below.

$$\eta = \frac{\sum w_{turbines} - \sum w_{pumps}}{q_B} \quad (25)$$

Equations 2,3,5 and 6 were utilized to find the total specific work of the turbines. Similarly, equations 8 and 10 were utilized to find the total specific work of the pumps. The specific heat of the boiler is equation 4.

Since thus far the values for enthalpies were all assumed to be isentropic, another set of equations was created to calculate the real enthalpies for the turbines and the pumps. This was done by applying the general equations of

$$\eta_{turbine} = \frac{h_i - h_f}{h_i - hfs} \quad (26)$$

$$\eta_{pump} = \frac{h_i - h_{fs}}{h_i - hf} \quad (27)$$

to every pump and turbine. Where $\eta_{turbine}$ and η_{pump} were given in the project description, through constraints five and six, as 0.85 and 0.90 respectively. To find the real specific entropy the variable h_f was solved for. At each state, where a pump or turbine was involved, the ideal specific entropy and real specific entropy were found.

The real efficiency should be less than the ideal efficiency which is less than the carnot efficiency. The carnot efficiency utilizes temperature boundaries and the equation is shown below.

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} \quad (28)$$

T_L is the lowest temperature of the system, which in this case is the ambient temperature of 25°C or 298.15 K. T_H is the highest temperature of the system, which is boundary temperature of the boiler, 495°C or 768.15 K. To calculate Carnot Efficiency, the units for temperature must be in Kelvins.

Matlab

Using the known values and the relations outlined above, a Matlab script was written to find the optimal pressures of the system. Enough information was given in the project information, that all of the unknown values could be solved for using XSteam. The first step was simply defining the different state properties using known values and relations within the Matlab script. These values were stored in arrays. Since a general range of the quality at each state was known, a check using the qualities was made to ensure that there were no errors in the code. Once it was decided that everything was running correctly, the quality checks were commented out. After all of the table values were accounted for, the C.o.E. equations and E.R.B. equations were coded into the script. The mass fraction values and the σs were created as matrices in order to verify legitimacy (i.e. $\sigma \geq 0.0$, etc.). Additionally, a filter was added to ensure valid values were making it through the program. In doing so, the entropy generated, $\sigma \geq 0$, the mass fractions were > 0, the pressures were descending in value $P_2 > P_3 > P_5$, and real efficiency (η) was $<$ Carnot efficiency. Since the pressures at state 2,3, and 5 are unknown, those will be the pressures that are iterated through. The pressures were bounded by the saturated pressure at state 7 and 80 [bar] as these are the minimum and maximum pressure values in the system. The code runs through with 100 values for each pressure variable. Implying, it would run a total of 100^3 or 1,000,000 iterations. After the code ran through all 1,000,000 possibilities, the largest η out of the data-set was found along with its position. The position was found in order to find the pressures. Like previously mentioned, the pumps and turbines were assumed to be isentropic, however this is not the case and the real enthalpy values were accounted for. The code for C.o.E. and E.R.B. was copy-pasted and variable names adjusted so that one set calculated the real efficiency and the other calculated the ideal efficiency. The code was then ran another 1,000,000 times and the final values were found.

RESULTS AND DISCUSSION

The table below is tabulated with the real values (i.e. using the isentropic efficiency to find specific enthalpy). A separate Matlab script was made to run through the optimized iteration, which provided the values below.

State	P [bar]	T [°C]	x [-]	h [$\frac{kJ}{kg}$]	s [$\frac{kJ}{kg-K}$]
1	**80.00**	**480.00**	Superheated	3349.5	6.66
2	4.11	144.60	0.96	2750.9	6.66
3	3.30	136.84	0.94	2628.6	6.66
4	3.30	**440.00**	Superheated	3358.8	8.11
5	0.88	254.52	Superheated	3040.1	8.11
6	0.074	40.00	0.98	2604.9	8.11
7	0.074	**40.00**	**0.0**	167.5	0.57
8	0.88	40.00	Subcooled	167.6	0.57
9	0.88	96.11	**0.0**	402.7	1.26
10	**80.00**	96.63	Subcooled	62.6	1.26
11	**80.00**	129.60	Subcooled	549.9	1.62
12	4.11	144.60	**0.0**	609.0	1.79
13	0.88	96.11	0.09	609.0	1.82

The final values for efficiencies were found as 31.08 % at P_2 = 4.11 [bar] , P_3 = 3.30 [bar] , and P_5 = 0.88 [bar] for the the ideal and 27.20 % at P_2 = 4.11 [bar] , P_3 = 3.30 [bar] , and P_5 = 0.88 [bar] for the real which is compared to the Carnot Efficiency of 61.12 %.

CONCLUSION

The professor assigned his Intro to Thermodynamics class with the challenge of optimizing the thermal efficiency of a traditional steam driven Rankine Cycle. Initially, the pumps and turbines were assumed to be isentropic. This was done to define some of the state properties. The remaining properties were found by utilizing Matlab and XSteam. To formulate equations that analyze the system, the 1^{st} and 2^{nd} law of thermodynamics were utilized. These equations got integrated into the Matlab script, allowing for iterations through different possibilities of the cycle. The initial assumption regarding pumps and turbines is disregarded and the isentropic efficiencies were used to find the real thermal efficiency. After numberous trials and errors, the iteration in which the system was optimized had an efficiency of 27.20 % when P_2 = 4.11 [bar] , P_3 = 3.30 [bar] , and P_5 = 0.88 [bar].

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